Frontier Topics in Empirical Economics: Week 1 Outline of Causal Inference

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¹College of Business, Shanghai University of Finance and Economics

September 9th, 2024

- Basic causal inference and statistical tools (Week 1-4)
 Potential outcome framework, RCT, matching vs regression, non-parametric method, machine learning, DAG framework
- IV (Week 5-7) IV, LATE, GMM, MTE, Bartik IV
- Causal inference with panel data (Week 8-9)

 Basic DID and event study, pre-trend testing, synthetic control, staggered DID
- Other topics (Week 10-13)
 RDD, Std err issues, Peer effect and spillover, intro to discrete choice model
- Student Presentation (Week 14-15)

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This is an example from Professor Chao Fu

- Consider a female labor participation problem
- Utility maximization of female *i*:

$$max \quad U_i(c_i, 1 - l_i) + \epsilon_{ij}$$

$$s.t. \quad c_i = w_i l_i$$
(1)

 c_i : consumption; l_i : labor supply; ϵ_{il} : unobserved taste shock; w_i : wage

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Assume that l_i is binary (work, not work)

$$I_i = 1 \text{ if } U(I = 1) \ge U(I = 0)$$

$$U_i(w_i, 0) + \epsilon_{i1} \ge U_i(0, 1) + \epsilon_{i0}$$
 (2)

lacksquare Then given w_i , we have a threshold value of $\epsilon_{i0}-\epsilon_{i1}$ for i to choose to work

$$l_i = 1$$
 if $\epsilon_{i0} - \epsilon_{i1} < \epsilon^*$

$$^* = U_i(w_i, 0) - U_i(0, 1)$$
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- lacksquare Assume that shock $\epsilon_{i0}-\epsilon_{i1}$ has a CDF $F_{\epsilon|w}$
- We have the following working probability for *i*

$$G(w) = Pr(I = 1|w) = \int_{-\infty}^{\epsilon^*} dF_{\epsilon|w}$$
$$= F_{\epsilon|w}(\epsilon^*(w))$$
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- 1. We can directly estimate probability function G with linearity assumption
 - \blacksquare Assume that G is a linear function

$$G(w) = \beta_0 + \beta_1 w_i \tag{5}$$

- lacksquare Linear Probability Model \Rightarrow We can use OLS to estimate eta
- This is called "Reduced-form" approach
- We usually identify it by some research "design" (IV, RDD, DID)
- Thus, it is also called "Design-based" approach

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- 2. We can estimate ϵ 's CDF F, and utility function U
 - We have the likelihood function as

$$L(\Theta^{U}, \Theta^{F}; data) = \prod_{i=1}^{N} F_{\epsilon}(\epsilon^{*})^{l_{i}} [1 - F_{\epsilon}(\epsilon^{*})]^{1-l_{i}}$$

$$(6)$$

- Θ^U is the parameter set of utility function; Θ^F is the parameter set of shock's CDF
- \blacksquare We use MLE to estimate Θ^U and $\Theta^F \Rightarrow$ Recover choice structure directly
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- Assume a linear utility function $U = \alpha w_i + \phi(1 l_i)$
- \blacksquare And ϵ follows T1EV distribution
- We have the likelihood function as:

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$$= \prod_{i=1}^{N} \left(\frac{\exp(\alpha w_{i})}{\exp(\alpha w_{i}) + \exp(\phi)}\right)^{l_{i}} \times \left(\frac{\exp(\phi)}{\exp(\alpha w_{i}) + \exp(\phi)}\right)^{1-l_{i}}$$
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- First, we need to clarify two important concepts
 - Internal validity
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 What would be the impact if we force all women to give birth to at least one child?
- The first one is "internal"
- The second and the third are "external"

- There are three layers of policy evaluation (Heckman and Vytlacil, 2007)
- Take One Child Policy (OCP) as an example
 - Evaluating the impact of a historical intervention What was the impact of the OCP on fertility rate?
 - Forecasting the impact of an intervention previously happened in environment A to happen in another environment B What would be the impact if we restart the OCP in 2023?
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- Target: Primitive parameters ⇒ Choice structure Agent's utility function, firm's production function, market structure...
- Advantages
 - Deeper economic thinking: we can understand the original decision-making processes.
 Great external validity → Solid under Lucas' critique.
 - More reliable counterfactual analysis
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- Individual treatment effect: $Y_{1i} Y_{0i}$
- Not available: There is only one world! Given i, you see either Y_{0i} or Y_1
- But we can consider averages: By differencing mean outcomes from the two groups

$$E[Y_{i}|D_{i} = 1] - E[Y_{i}|D_{i} = 0]$$

$$= \underbrace{E[Y_{1i}|D_{i} = 1] - E[Y_{0i}|D_{i} = 1]}_{\text{Average Treatment on the Treated (ATT)}} + \underbrace{E[Y_{0i}|D_{i} = 1] - E[Y_{0i}|D_{i} = 0]}_{\text{Selection bias}}$$
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- ATT: Causal effect on the treated group
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Randomization can solve the selection problem

Assume that we randomly assign the treatment to the population

$$\mathcal{D}_i \perp \!\!\!\perp Y_{0i}, Y_{1i} \tag{10}$$

■ Then we have selection bias to be zero

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 Thus, simple difference between the mean of treated and untreated group is ATT (and overall ATE)

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Conditional Expectation Function (CEF)

■ CEF is the conditional expectation of an outcome Y_i , given some predictor vector X_i

$$E[Y_i|X_i=x] = \int tf_y(t|X_i=x)dt$$
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where f_v is pdf

- This is a population concept $(n \to \infty)$
- It describes a prediction of X on Y, but NOT necessarily causa
- \blacksquare We can always decompose Y_i as predicted part (CEF) + error particles

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- \blacksquare CEF is the best predictor of Y_i given X_i
- It minimizes the mean squared prediction errors
- Let $m(X_i)$ be any function of X_i . The UEF solves
 - $E[Y_t|X_t] = argmin_{m(X_t)}E[(Y_t m(X_t))^*]$
- so it is the MMSE predictor of Y_i given X_{j+1}

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Theorem 3.1.2 in MHE

Let $m(X_i)$ be any function of X_i . The CEF solves

$$E[Y_i|X_i] = argmin_{m(X_i)}E[(Y_i - m(X_i))^2]$$

so it is the MMSE predictor of Y_i given $X_{i,z}$

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Regression is a linear prediction that minimizes the mean squared error

$$Y_i = X_i^I \beta + \epsilon_i$$

 $\beta = argmin_b E[(Y_i - X_i^I b)^2]$

We have the first order condition (moment condition) as

$$E[X_i(Y_i - X_i'\beta)] = 0$$

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Tips: Difference between β and $\hat{\beta}_{OLS}$

$$\beta = E[X_i X_i']^{-1} E[X_i Y_i]$$
$$\hat{\beta}_{OLS} = (X'X)^{-1} X'Y$$

- $m{\beta}_{OLS}$ is an estimator of $m{\beta}$ (there can be alternative estimators, e.g. MLE)
- Population vs Sample, Identification vs Estimation
- \blacksquare X_i is an $1 \times k$ vector, Y_i is a scalar. They are random variables
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Under this randomization, CEF is linear, then

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Key to go from correlation/prediction to causality: Conditional Independent Assumption (CIA)/Selection on Observables

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- For each $X_i = x$, we have the following regression

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Main takeaways from this par

- Strength of assumptions regarding unobservable e
 Causal model (CIA) > CEF (Mean Independence) > Linear regression (Uncorrelated)
- CEF is the best predictor of Y given X
- Linear regression is the best "linear" predictor of Y given X
- Linear regression is the best linear approximation of CEF
- Under CIA and homogeneous TE, regression coefficient is the TE
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- Consider two treatments A and B for COVID
- We examine the effect of the treatments by patients' conditions (mild/severe
- We have the death rate by treatments and conditions as

- Total death rate: A < B</p>
- Death rate within condition group: A > B

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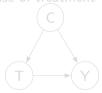
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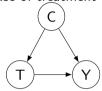
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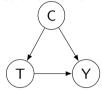
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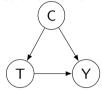
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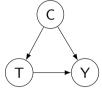
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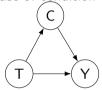


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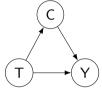
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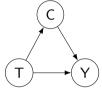
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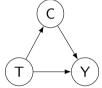
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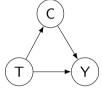
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- To have CIA, we need a lot of controls:

 Gender, race, nationality, birth weight, IQ, parents' education, parents' income....
- \blacksquare Curse of dimensionality: There are too many dimensions in X_i
- lacktriangle We will not have enough observations for each value of X_i to estimate $\hat{\delta}_j$
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References

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